DESIGN AND ANALYSIS OF CENTRALLY SUSPENDED CAGE-LESS DIFFERENTIAL

SURAJ ARU[1], PRAVIN JADHAV[2], SHUBHAM GAJABHIYE[3], VINAY JADHAV[4] & PRATIM ANGANE[5]

[1]Department of Production Engineering & Industrial Management, College of Engineering, Pune, Maharashtra, India

[2,3,4,5]Department of Mechanical Engineering, College of Engineering, Pune, Maharashtra, India

ABSTRACT

Cornering fluency is a prime factor determining the performance of a vehicle. Apart from suspension design effectiveness, a differential used in power train enhances the fluency in cornering. With many advancements coming up in automobile sector and the various inventions made, a cage-less differential is one such development. Reduction in weight and space, due to its compactness, makes it a vital device. In this study, a cage-less differential was designed and analysed for its use on an All-Terrain Vehicle (ATV). The entire design of differential gears was done theoretically using Buckingham's equation, and was then simulated in Kiss Soft 03-2013. The modelling part was carried out in CATIA V5R20 and analysis was done using ANSYS Workbench 14.0. A sequential approach of gear design, cross-pin design and bearing design was adopted. Its use on an All-Terrain Vehicle was seen to be effective.

KEYWORDS: Power Train, Open Differential, Cage-less, Gear Analysis, CATIA, ANSYS 14.0

INTRODUCTION

A differential is a mechanical device used to split the final drive to the wheels of a vehicle. This split enables the two wheels to rotate at different angular velocities which is required while moving around a corner. When a vehicle goes around a corner, the outer wheel covers a greater distance than the inner wheel. This will happen only when the outer wheel travels at a greater rotations per minutes than the inner wheel.

A differential consists of a crown gear, planetary gears, sun gears and a cross-pin. The drive to crown gear comes from the gearbox output shaft. The inner part of the crown gear consists of a housing which holds the set of sun and planets gears. The planet gears are connected through a pin and the sun gears are in mesh with the planet gears. The final output through a differential comes from the sun gears. Each of the sun gears has a drive shaft which provides drive to the wheel. For designing a differential, the maximum torque and rotations per minute are of prime importance. The torque transmitted also determines the number of planet gears in the housing, with more number of gears for higher torque and less number of gears for lower torque.

The differential designed in this study comes without the housing inside the crown gear. This results in weight and space reduction and makes the differential compact. This differential was designed for an ATV made for a competition called BAJA. BAJA is an annual event held by Society of Automotive Engineers (SAE). In order to achieve a better acceleration, it is very important to keep the weight as low as possible. The strength, reliability and weight should be optimized to the best possible outcome.

PRE-REQUISITES

The different parameters used for design and analysis of differential were selected considering an off road vehicle

powered by 10 hp engine, 305 cc capacity. Also, it is considered that the differential is coupled with a gearbox which effectively provides a gear ratio of 36. The various other important parameters are stated below.

Gross Vehicle Weight = 240 kgs

Weight Distribution = Front – 96 kgs, Rear – 144 kgs

Wheel Track = 49 inch

Engine Power = 10 hp

Engine Torque = 19.2 Nm @ 2700 rpm

Engine Maximum RPM = 3800 rpm

Tyre Size = 23 x 7 x 10 inch

Gear & Pinion Material = 20MnCr5, Yield Strength: 850MPa and Ultimate Strength: 1300MPa (Post Hardening)

Mounting Plate Material = Mild Steel, Yield Strength: 235MPa and Ultimate Strength: 340MPa

Hardness of Gears = 600 BHN

DESIGN OF A CENTERED CAGELESS DIFFERENTIAL

The design calculations of the various parts used in the differential are as follows:

Parts Used in Differential

- Ring gear

- Bevel gear

- Cross pin

- Mounting plate

Ring Gear Design

While designing crown gear, the following flow is adopted. Gears are designed by using Buckingham equation. Manual design calculation is as follows:

$$Pt = 2 \times \frac{Mt}{D}$$

Where,

Pt = Tangential load on pinion (N)

Mt = Torque transmitted by crown gear (Nmm)

D = Pitch circle diameter (mm)

Buckingham equation is:

$$Pd = \frac{21v(Ceb + Pt)}{21v + \sqrt{Ceb + Pt}}$$

Where,

Pd = Dynamic load (N)

v = Pitch line velocity (m/s)

C = Deformation factor (N/mm^2)

e = Error (mm)

b = Face width (mm)

Peff = Cs x Pt + Pd (1)

Where,

Peff = Effective Load (N)

Cs = Service factor

The maximum permissible beam strength of gear tooth is given as follows:

Sb = m x b x σb x Y (2)

Where,

Sb = Beam strength of gear (N)

σb = Permissible bending stress (N/mm^2)

Y = Lewis form factor

m = Module (mm)

$$\sigma b = \frac{Sut}{3}$$

Where,

Sut = Ultimate tensile strength (N/mm^2)

The wear strength of gear tooth is as follows:

Sw = b x Q x dp x K (3)

Where,

Sw = Wear strength of gear (N)

dp = Pitch circle diameter of pinion (mm)

Q = Ratio factor

K = Material constant

$$Q = 2 \times \frac{Zg}{(Zg + Zp)}$$

Where,

Zg = Number of teeth of gear

Zp = Number of teeth of pinion

$$K = 0.16 \left(\frac{BHN}{100}\right)^2$$

Where,

K = Load stress factor

BHN = Brinell hardness number

To avoid gear tooth breakage and wear failure of gear tooth following condition must be satisfied

$Sb \geq Peff$ and $Sw \geq Peff$

Solving equation 1, 2 and 3 above condition is satisfied.

Therefore the design is safe. The values obtained by performing the above calculations are as shown in table 1:

Table 1: Crown Gear Specifications

Parameter	Values
Gear type	Spur gear
Pressure angle	20 degree
Number of teeth	62
Torque transmitted	700 Nm
RPM	110
Face width	15 m

Bevel Gear Design

While designing bevel gears following flow is adopted

$$Ptp = \frac{2\,Mtp}{Dm}$$

Where,

Ptp = Tangential load on pinion(N)

Mtp = Torque transmitted by pinion(Nmm)

$$Rm = \frac{Dp}{2} - \frac{b \times \sin(\gamma)}{2}$$

Where,

Rm = Mean radius (mm)

b = Face width (mm)

γ = Pitch angle (degree)

Dp = Pitch circle diameter of bevel pinion

Assuming, $\frac{b}{A_o} = 0.3$

$$A_o = \sqrt{\left(\frac{Dp}{2}\right)^2 + \left(\frac{Dg}{2}\right)^2}$$

Where,

Ao = Cone distance (mm)

Dg = Pitch circle diameter of bevel gear

Permissible beam strength of the tooth is given by,

$$Sb = m \times b \times \sigma b \times Y \times \left(1 - \frac{b}{A_o}\right)$$

Wear strength of tooth is:

$$Sw = 0.75 \times b \times Q \times Dp \times \frac{K}{\cos Y}$$

Where,

γ = Pitch angle

Effective load on tooth is calculated by using Buckingham equation.

$$Pd = \frac{21v(Ceb + Pt)}{21v + \sqrt{Ceb + Pt}}$$

Peff = Cs x Pt + Pd

To avoid gear tooth breakage and wear failure of gear tooth following condition must be satisfied

$$Sb \geq Peff \quad And \quad Sw \geq Peff$$

Solving equation above condition is satisfied.

Therefore the design is safe. The values obtained by performing the above calculations are as shown in table 1:

Table 2: Bevel Gear Specifications

Parameter	Values
Pressure angle	20 degree
Number of teeth on planet	10
Number of teeth on sun	18
RPM	110
Face width	10.5 mm
Module	3.5 mm

Cross Pin

Cross pin is designed considering the shaft design.
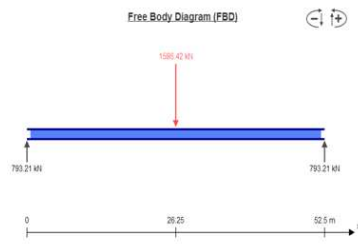
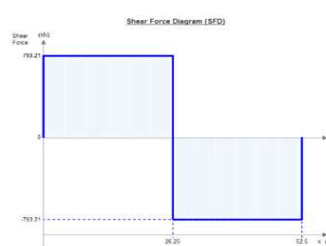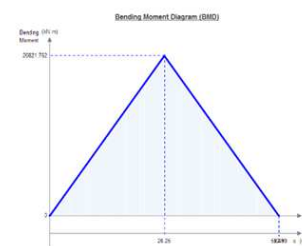**Figure 1: Free Body Diagram**   **Figure 2: Shear Force Diagram**   **Figure 3: Bending Moment Diagram**

The horizontal part of the pin is considered as simply supported beam.

Diameter of shaft is calculated using ASME code for shaft design.

$$d^3 = \frac{16}{\Pi\tau} \sqrt{(Mb)^2 + (Mt)^2}$$

Where,

d = diameter of shaft

τ = Maximum shear stress

Mb = Maximum bending moment

Mt = Torsional moment

Mounting Plate

The mounting plates are used to constained the lateral movement of the cross pin. This plate is to be mounted on the ring gear by means of two M6 bolts.

CAD MODELLING

Catia V5R20 parametric CAD solid modelling software was extensively used for the generation and analysis of the differential and driveline components. On the basis of dimensions of differential components, each component is modelled in CAD modelling software Catia V5R20 having images as shown below:

Final Design

Cross Pin

Quantity: 1
This part connects the bevel sun gears and transmits power from the ring gear to the bevel sun gears.
Material: 20MnCr5

Figure 4: Cross Pin

Ring Gear

Quantity: 1

This part houses the planetary gear set of bevel gears and transmits power from gearbox to wheels

Material: 20MnCr5

Figure 5: Ring Gear

Sun Bevel Gear

Quantity: 2

These gears connect to the output shafts of the differential and transmit power to the wheels.

Material: 20MnCr5

Figure 6: Sun Bevel Gear

Planet Bevel Gear

Quantity: 2

These gears are mounted on the Cross pin and provide force to drive the side gears while allowing them to spin at different speeds when the car is negotiating turns or is stuck in mud.

Material: 20MnCr5

Figure 7: Plant Bevel Gear

Mounting Plate

Quantity: 2

These plates fix the pin into the crown gear and avoid its lateral movement.

Material: Mild Steel

Figure 8: Mounting Plate

M6 Bolts

Quantity: 2

These bolts are used to restrict the lateral movement of the cross pin, out of the ring gear.

Material: Hardened Steel

Figure 9: Bolts

Figure 10: Exploded View of Differential   Figure 11: Assembled View of Differential

STATIC ANALYSIS

A simple structural analysis was performed as the first step to see if components were structurally strong. If a component failed with the loadings, then no need to continue stress or fatigue analysis since the component is not strong enough to be used. The analysis of the various components of the differential was done in ANSYS 14.0 WORKBENCH for meshing as well as solving.

Meshing of all the parts was done in ANSYS. The mesh is generated by using tetrahedron elements of 1 mm size. Mesh quality is further improved by using proximity and curvature function. This improves mesh density where curvature is small or edges are closed in proximity.

Ring Gear

Gear teeth are subjected to both bending and wear. The section where it experiences the maximum stress is the root of the tooth. It is considered that, there are 3 teeth in contact with the mating gear. Tangential force (Ft) because of the torque which the gear is transmitting is applied on these three teeth. The total tangential force is distributed amongst the three teeth considering one tooth takes 50% of force and others take 25% each. Another force acting on the gear is Radial forces (Fr) which is generated due to separating force between the two meshing gears. This force acts towards the centre of the gear. Magnitude of force (Ft) and (Fr) acting on gear are taken from above designed values.

The constraints which were given are only physical constraints. The inner surface of the gear is fixed, where shaft is mounted and also the diagonally opposite surfaces of the slots which are made for fixing the pin.

The maximum stress produced in the gear is **254.21MPa** and the maximum deformation is **0.0136mm** which shows that the gears are safe.
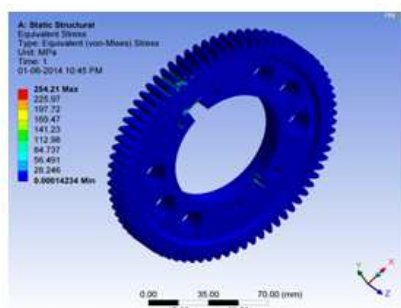

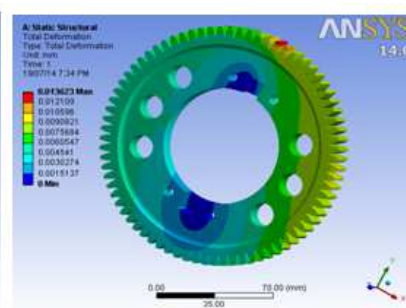**Figure 12: Stress Analysis in ANSYS   Figure 13: Maximum Deformation in ANSYS**

Bevel Sun Gear

When designing a pair of bevel gears there are 3 forces which come into action. These are radial force (Fr),

tangential force (Ft) and axial force (Fa). To analyse the bevel gears these three forces have been applied on gear's teeth. It is considered that there are 2 teeth in mesh with the bevel pinion.

After analysis in ANSYS it is found out that the maximum stress zone is at the root of the gear near the small filleted areas. The maximum stress in the component is **426.07 MPa.** The displacement of the gear is near the tops of the loaded teeth. The maximum displacement in this component is **0.0385 mm.** From the values of stress and displacement it is shown that the given component is safe.
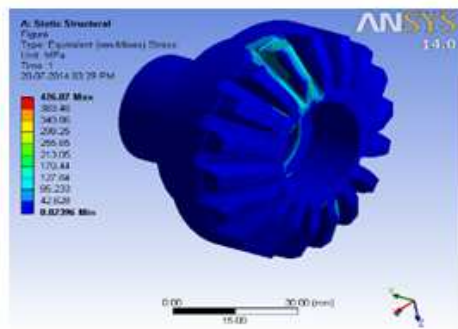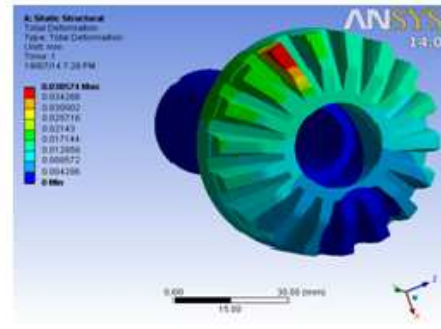

Figure 14: Stress Analysis in ANSYS   **Figure 15: Maximum Deformation in ANSYS**

Cross Pin

Cross pin consists of two different diameters. The smaller end of pin was constrained at its ends excluding the pin end surface and a torque was applied at the location where it was in contact with the planet bevel gears. The larger end of the pin is simply supported on the journal bearings fitted inside the sun bevel gear. The compressive force is applied at the end face of the pin and rest all faces are fixed. This compressive force is calculated from the radial force on the ring gear.

The results of ANSYS show that the maximum stress generated in the component is at the fillet edges as shown in the below figure. The maximum stress is of **519.63 MPa** and maximum deformation of the pin is **0.0041 mm.** Both these values are within the safety limit.
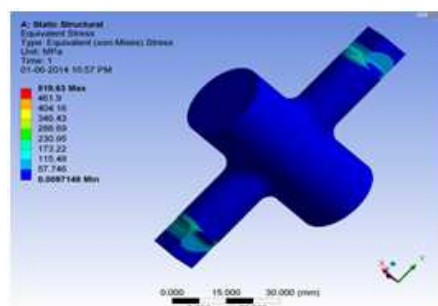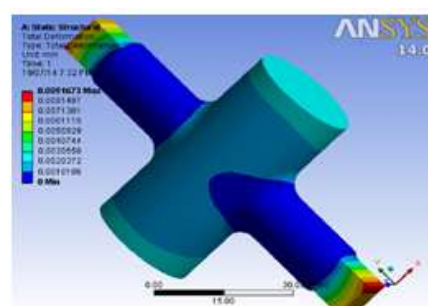

Figure 16: Stress Analysis in ANSYS   **Figure 17: Maximum Deformation in ANSYS**

Planet Bevel Gear

The planet gears are mounted on the cross pin and drives the bevel sun gears when the differential is in working condition. These gears are constrained at their bore and the forces which bevel sun gear transmit are also applied to this gear. It is constrained on the inner cylindrical face and the forces are applied on two teeth. These forces are calculated by considering that two teeth are in contact with bevel sun gear.

It is found that the maximum stress occurs at the root of the tooth of bevel gear. The value of maximum stress produced is **175.67 Mpa** and the maximum deflection in this part is **0.0095 mm.**
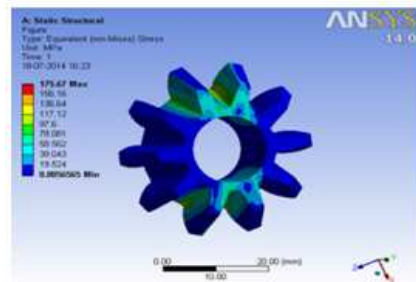

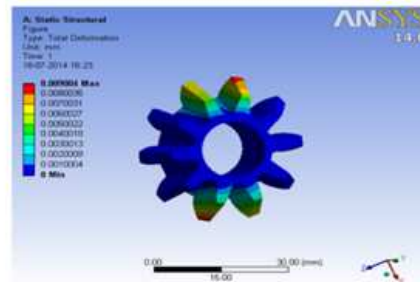Figure 17: Stress Analysis in ANSYS   Figure 18: Maximum Deformation in ANSYS

RESULTS AND DISCUSSIONS

- Overall, this project is a success. Our goal is to design a lightweight cage less open differential, it is much more compact and better in strength perspectives.

- Gear ratio and face width varies both Strength and maximum stress on Gear surface.

- Stresses and Deformation obtained are within the permissible range to avoid static and fatigue failure.

- Minimum factor of safety is kept more than 1 but as low as possible to avoid unnecessary weight of Components which increases weight of the differential and in total of vehicle.

- Permissible stress can be determined by the properties of material. Actual stress can be determined by modelling and finite element analysis.

- The fatigue life of the components seems to be effectively high with the exception of the gears.

This differential is easy to assemble and disassemble. For an auto racing application, where the driveline is routinely disassembled, inspected and repaired before and after race. One of the major challenges faced in this project was to find the gear sizes that would be strong enough, while still keeping the overall assembly as small as possible.

The differential designed in this study has a total mass of 3.44 kg which equals to a weight reduction of 37.45% as compared to a caged differential. Thus, there is a significant decrease in the mass of the component.

CONCLUSIONS

This paper has illustrated the entire design methodology of a cage less differential. The efforts taken to account for all the necessary design and analysis considerations have provided a solid starting point into the fundamentals of differential. Finite element analysis was carried out on CAD models of an efficient and potential design. Using the FEA output, a design was created that met all the stipulated functional requirements and possessed a reasonable factor of safety while still saving significant mass and rotational inertia as compared to the other conventional differentials.

ACKNOWLEDGEMENTS

REFERENCES

1. "Design of machine elements" by V. B. Bhandari Second edition, Tata Mc Graw Hill, Spur gear page 653 to 678, 720 to 722 "Bevel gear" page 720 to 739.

2. Zeping Wei Stresses and Deformations in Involute Spur Gears by Finite Element Method, Saskatoon, 2004.

3. Smith, Carroll (1996). Drive to Win. Carroll Smith Consulting Inc.

4. Saleem Merkt & Terrence Gilbert Electronic Differential and Hybrid Power train Design for NCSU Formula Hybrid North Carolina State University 2009

5. Design and Structural Analysis of Differential Gear Box at Different Loads "International Journal of Advanced Engineering Research and Studies "

6. Anngwo Wang, Computerized Design and Generation of Spiral Bevel Gears and Forged Straight Bevel Gears with Improved Geometry, Chicago: University of Chicago, 1997